

**SPHERICAL ASTRONOMY TEST**  
**Sep 27, 2017**

1. Here's a geometric problem for warm-up: given that we have just witnessed an awesome solar eclipse, compute what is the longest possible duration of an eclipse. The radius of the Sun is 696,000 km, the radius of Earth is 6378.1 km, the semi-major axis of Earth's orbit is 149.6 million km, and orbital eccentricity is 0.0167. The radius of the Moon is 1738.1 km, the semi-major axis of its orbit around Earth is 384,400 km, and orbital eccentricity is 0.0549. The average length of 1 month is 29.5 days. If you had forgotten an equation for the ellipse, here it is:

$$r(v) = \frac{a(1 - e^2)}{1 + e \cos v},$$

where  $v$  is the angle measured from perihelion, i.e. Earth's closest approach to the Sun.

2. This class' favorite star is undoubtedly Deneb (right, Casey?). Its equatorial coordinates are  $\alpha = 20^{\text{h}} 41^{\text{m}} 26^{\text{s}}$  and  $\delta = 45^{\circ} 16' 49''$ . We are observing tonight (Sep 27, 2017) from Villanova ( $\varphi = 40^{\circ} 02' 14''$  N,  $\lambda = 75^{\circ} 20' 57''$  W).
  - a) At what time(s) will Deneb be seen at  $60^{\circ}$  above the horizon?
  - b) At what altitude will Deneb be seen when it crosses the E-W meridian?
  - c) On what date in 2018 will Deneb culminate at 9:30pm?

Neglect any effects of refraction or precession.

3. Redo problem (2), but this time take refraction and precession corrections into account. Assume that the equatorial coordinates of Deneb are given in the epoch 2000.0.
4. Two watchtowers at known geographic coordinates  $(\varphi_A, \lambda_A)$  and  $(\varphi_B, \lambda_B)$  detect a distress radio signal from a ship at unknown geographic coordinates  $(\varphi_S, \lambda_S)$ . Each watchtower measures the angle w.r.t. geographic north at which the signal is received; denote these angles with  $\delta_A$  and  $\delta_B$  for watchtower A and B, respectively. What are the unknown coordinates of the ship? Fun fact: this method is known as *triangulation*.

Test time: 150 minutes. The best of luck, ladies and gents, make me proud!