THE BOUNDARY VALUE PROBLEM

Due date: 2/7/2024, 9am

In our previous assignment we explored the *initial* value problem: given the system of first-order ordinary differential equations $\mathbf{y}'(t, \mathbf{y})$ and the corresponding initial values $\mathbf{y}(t = 0)$, we computed the parameter values $\mathbf{y}(t)$ at some later time t. This is a fully determined problem and, while differential equations might be misbehaving, a numerical solution (unstable as it may be) is unique and always exists. This week we will focus on the *boundary* value problem: we are interested in initial values that result in a boundary condition that we impose on the solution. The problem is no longer (necessarily) unique and the solution is found iteratively, by converting the problem to iterative minimization.

a) Imagine shooting a rocket with speed v an angle ϕ with respect to the horizon. Find the initial conditions for which the rocket will be at a height h and distance D after time t.

b) Solve the problem numerically, using a Runge-Kutta integrator by first guessing, and then iteratively solving for initial values. Then you can add air resistance and other bells and whistles.

c) Explore the concept of gravity assist. Say a spacecraft of negligible mass is moving past a massive celestial body (planet, moon, . . .). How does its velocity, kinetic energy and potential energy change after the encounter? Study the dependence of the impact factor, angle of incidence and velocity ratio. Make this an isolated system, i.e. there are no other bodies that contribute to the force field. Solve this in a stationary frame (relative to the planet and the satellite).

d) Now take this to the next level: launch a rocket from Earth towards the Moon and determine initial conditions that will give the rocket the maximal gravity assist by the Moon. Plot the family of trajectories around the optimal trajectory; how sensitive is "the final destination" to your initial conditions?