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A Computational Universal Curvature Fitting Algorithm

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ABSTRACT

With the current power of modern computers, computational modeling and fitting has become the de facto choice in the analysis of astrophysical data. Here I present CURVFAM (CURVature Fitting AlgoritM), a code that models a universe under a prescribed curvature metric. Unlike current universe simulators, this code is optimized for solely curvature modeling to conserve time and resources. Future implementations of this code will allow for the determination of universal curvature based on real world observations. This paper gives an overview of the scientific reasoning and formulations behind the program as well as a look into what the future of CURVFAM will offer.

1. INTRODUCTION

The measure of an object's flatness is solely a question of the object's geometry. An object can have one of three types of curvature that can have (in cases of extreme curvature) significant effects on measurements. The three types of curvature are: flat (zero curvature), open (negative curvature), and closed (positive curvature). On terrestrial scales, an object's curvature can easily be measured by observing the effects of two initially parallel lines and the sum of the interior angles of a triangle. On a flat (euclidean) surface (e.g. a table top), parallel lines will forever run equidistant from each other and never intersect while the interior angles of a triangle will add up to 180°. On a closed surface (e.g. the surface of the Earth), lines that start parallel will eventually intersect and the interior angles of a triangle will add up to greater than 180°. On an open surface (e.g. a saddle), parallel lines will eventually diverge and the interior angles of a triangle will add up to less than 180°.

In an attempt to determine the shape of the universe and how it might have evolved over time, some groups of researchers have taken to observing the equations that govern the shape of the universe, the Friedmann–Lemaître-Robertson–Walker (FLRW) metric, and analyzing the possible universal results (Vardanyan et al. 2011; Yu & Wang 2016; Mortsell & Jonsson 2011). Attempts range from directly attempting to solve the equations (Odintsov & Oikonomou 2019; Steiner 2007) to analyzing many different models using statistics (Vardanyan et al. 2011).

The two main methods used for the determination of the curvature parameter K use supernovae and the cosmic microwave background (CMB). The first method involves comparing data from supernova surveys with solutions to the FLRW metric across various tested values for the density parameters of matter Ω_M and dark energy Ω_Λ (Inserra et al. 2021; Wang et al. 2005). Alternatively, other studies utilize power spectra of the irregularities in the CMB to statistically estimate curvature (Vardanyan et al. 2011). While each method of determining the density parameters yields plausible results, there is still tension in the ultimate determination of the global curvature of the universe between the various methods (Handley 2021).

The method of prescribing different models to the universe and statistically analyzing which fits observed qualities of the universe better is typically used for constraining values such as the Hubble constant (H_0), the current matter density parameter (Ω_0), and the current dark energy density parameter (Ω_Λ) to name a few (Inserra et al. 2021; Gupta 2019). It can, however, be extrapolated further to analyze additional parameters or even the shape of the universe as a whole.

A new approach to determining the global structure of the universe has been gaining popularity with the ever increasing processing power of modern computers. This approach models an entire universe of particles and subjects them to the laws of physics allowing for the system to evolve with time. At first, such simulations (called N-body simulations) were used to observe galaxy clustering (Aarseth et al. 1979), but more recently, these simulations are modeling universes with trillions of particles to high degrees of accuracy (Maksimova et al. 2021). The implications of such detailed simulations is the ability to directly measure the desired parameters ($\Omega_M,\ \Omega_\Lambda,\ H_0$) from the simulated universe.

The difficulty with the universal simulations is the amount of computational power and time required to process such large amounts of calculations. This makes the use of such simulations excessive for solely determining curvature parameters. In the case of curvature modeling, a simulation more specifically designed with curvature in mind might provide a more appealing option.

In this paper I present a new method for determining the global curvature of the universe. I introduce the logical framework and baseline code for a curvature fitting algorithm (CURVFAM). I summarize the theory behind this method in Section 2. In Section 3 I explain a lower dimension model as a simpler introduction to the more general case. I follow this with a more comprehensive model in Section 4. Future addendums to the code as well as streamlined processes are discussed in Section 5 followed by a concluding summary in Section 6.

2. THEORY

There are two overarching methods to determining the properties of the universe: a bottom-up approach and a top-down approach. In a bottom-up approach, observable values and parameters are measured from objects existing in the universe (e.g. supernovae and quasars). A theory is then applied to the observables to determine the properties of the universe. In a top-down approach, the properties of the universe are assumed to be known and a value or function of values are applied to them to determine what values the observables would take on.

Here, I apply a top-down approach to determine a curvature function for the observable universe. I start by assuming that the curvature metric (K) can change as a function of both time and space.

$$K(\vec{r}, t) = \Upsilon(\vec{r})T(t) \tag{1}$$

By prescribing a known curvature metric to a universe of test data, observables can be measured with respect to a chosen reference point. In this case, the observables will be: perceived distance to any object from the reference point and the measured angle formed between two observed objects with respect to the reference point.

The important distinction to make for observations in curved space is that the distance between any two objects is going to change depending on the curvature. As such, the apparent angle between the two objects (from the reference point) is going to increase or decrease based on the curvature. Figure 1 shows how objects that would be in line in euclidean space (dashed lines) would be perceived to be separated by a different angle in curved space.

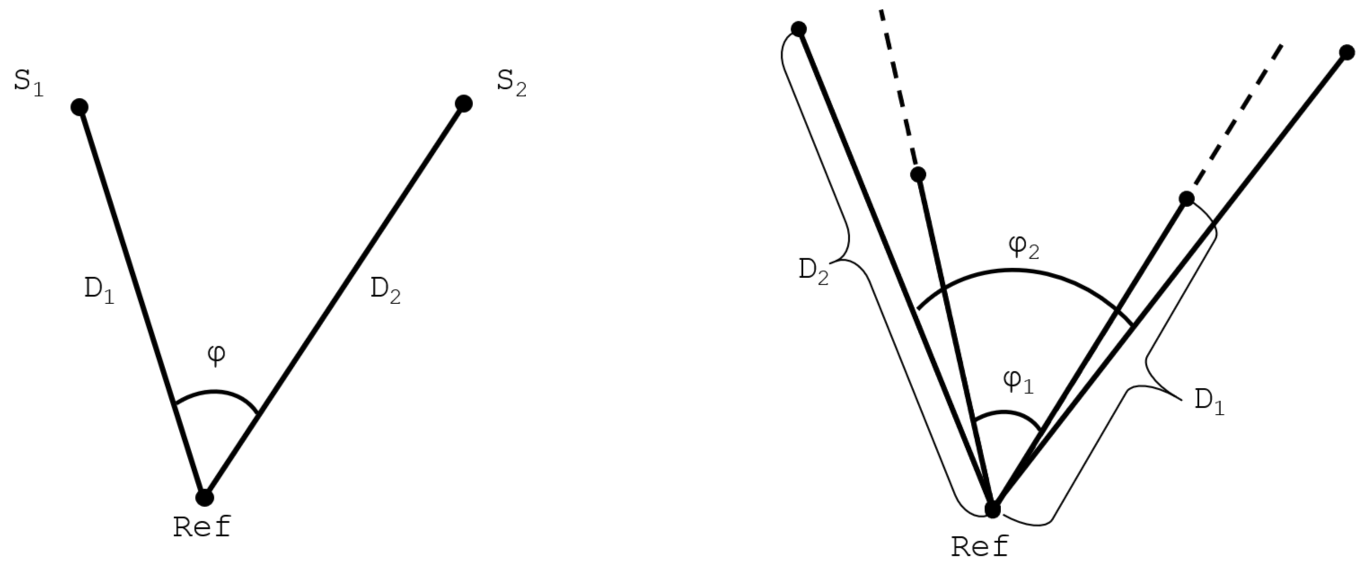

Figure 1. Left: The observables measured from a reference point (distances: D_1, D_2 angle: φ). Right: The difference between true positions of objects in line (dashed line) and their perceived locations in curved space (solid line).

By determining how the observables change for a set of objects with known, fixed locations, the dependence of observables on the curvature is shown. With the top-down model constructed, the curvature function (Equation 1) can be optimized for a curvature that yields observables that we see in our own universe.

3. 1-D MODEL

Before designing a model that incorporates all dimensions, I first apply the math to a simpler one-dimensional model to see how curvature affects the observable on geometries that are easier to both comprehend and simulate.

For a one-dimensional model, the only observable would be the distance from the reference point along the direction of space. In euclidean space, this model would be a straight line. The curvature metric in this model is a function position. To model this, I generate a set of uniformly distributed points in flat space, then generate their positions in curved space by applying a prescribed curvature function to their locations. Figure 2 shows how the positions of the objects in flat space are transcribed as new positions in curved space.

For any one-dimensional space, the distance between any two points is given by the distance formula for curves. Equation 2 shows the distance D_i to any object S_i relative to the reference point. Here $|K'|$ represents the modulus of the derivative of the curvature function.

$$D_i = \int_0^{S_i} \sqrt{1 + |K'|^2}\, dx \tag{2}$$

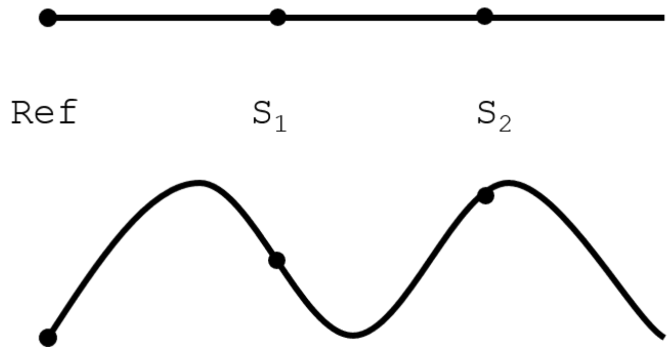

Figure 2. How one-dimensional distance changes as the curvature of the space changes. In flat space the distance traveled is a straight line, but in curved space the distance traveled is a path length along the curvature function.

The effect of curvature on the distributions of distances is clear when the 1-D universe is populated and given an extreme curve in the form of a polynomial. Figure 3 displays how the distribution of distances changed from uniform spacing and density to a less uniform distribution when acted on by curvature.

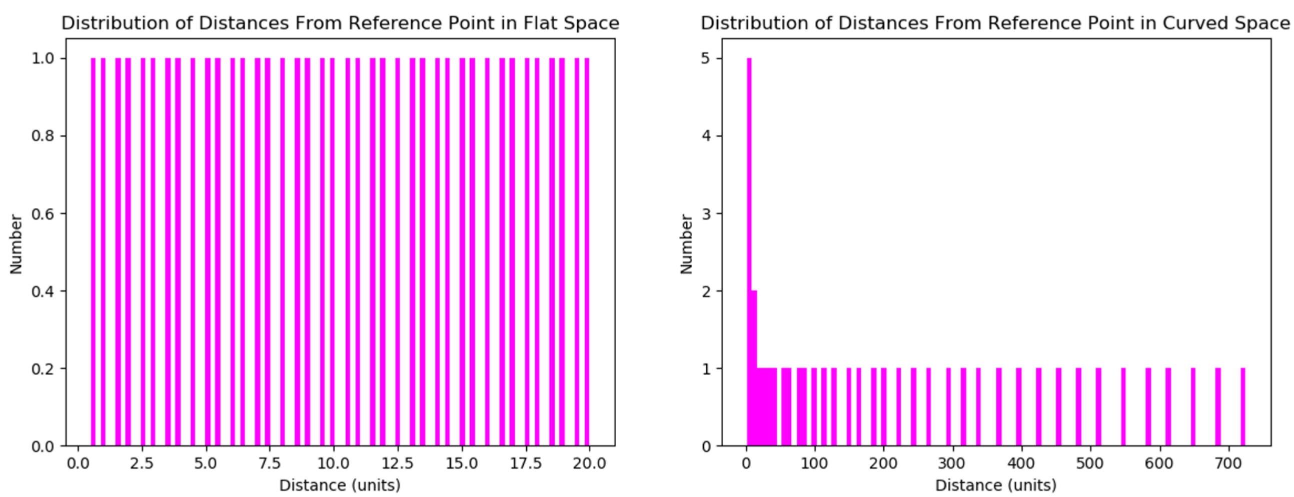

Figure 3. How the distribution of one-dimensional distances changes when curvature is applied. It is important to note that the exact results depend on the curvature functions used.

4. COMPREHENSIVE MODEL

In transitioning to higher spatial dimensions, it is important to note that the reference point is no longer at the zero point of a function of curvature. Rather, the reference point is now at the origin of a three-dimensional universe that is uniformly populated with points. The universe starts flat with the points distributed homogeneously and isotropically throughout the volume. Then, a curvature metric is prescribed to the space. Unlike the one-dimensional case, the curvature metric here is a series of three equations, each describing the curvature of a different spatial dimension. Equation 3 shows how the curvature metric is broken into three separate functions where K_i describes the curvature of space in the i^{th} dimension. Spherical coordinates (φ, θ, r) are chosen due to the fact that all observations are relative to a single reference point which I consider the origin here.

$$\begin{aligned} K_\varphi(\vec{r}, t) &= \Upsilon(\vec{r})T(t) \\ K_\theta(\vec{r}, t) &= \Upsilon(\vec{r})T(t) \\ K_r(\vec{r}, t) &= \Upsilon(\vec{r})T(t) \end{aligned} \tag{3}$$

The prescribed curvature metric is applied to each point's position vector resulting in a new primed position vector that gives the point's primed location in curved space with regard to the euclidean space. Note here that while the r component of the spherical coordinates is representative of the radial distance from the reference point, this is synonymous with time due to the finite speed of light. Figure 4 shows how curvature causes the observed spatial distribution of points to warp.

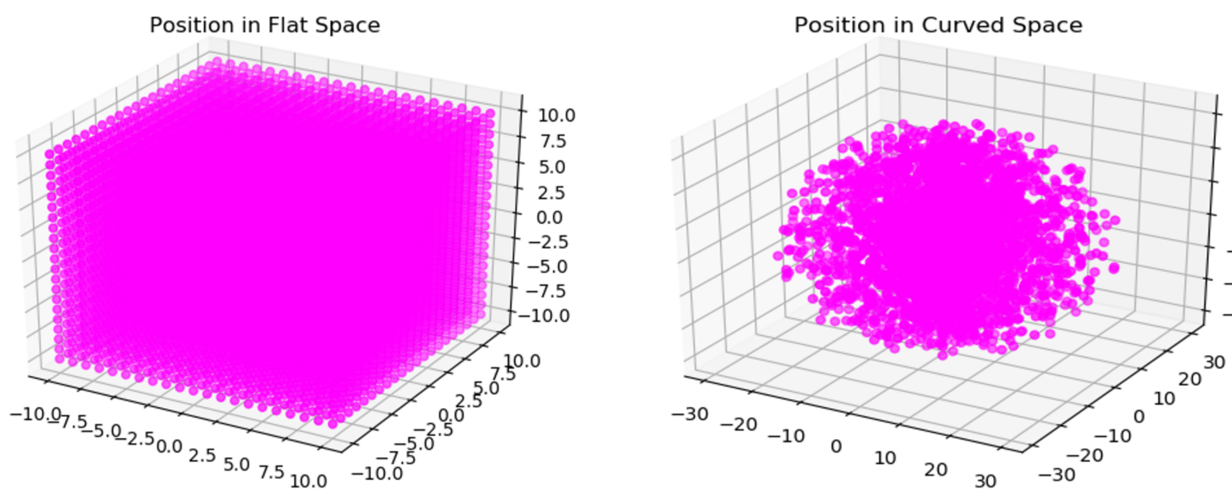

Figure 4. How the three-dimensional uniform distribution is affected by spatial curvature. The reference point is at the center of the distribution. It is important to note that the exact results depend on the curvature functions used.

The degree of the effect is determined by the exact curvature that is applied to the model. In these preliminary cases, the curvature parameter is simply a series of polynomials to display more radical and clear results. When fitting observational data, a series of exponentials with variable coefficients and orders would be more descriptive.

The change in spatial distribution also changes the observed angle between any two points. To determine the effects of curvature on the observed angles, I considered every pair of two adjacent data points. Adjacent is defined here as two points that, in flat space, lie directly next to each other along a single dimension (x, y, z) and differ by one separation distance in only one coordinate (eg. (1,1,1) is adjacent to (1,2,1) but is adjacent to neither (1,2,2) nor (1,3,1)). I use the angle distribution and the average angle value for flat space as a reference against which to compare the distribution and average angle value for the same pairs in curved space.

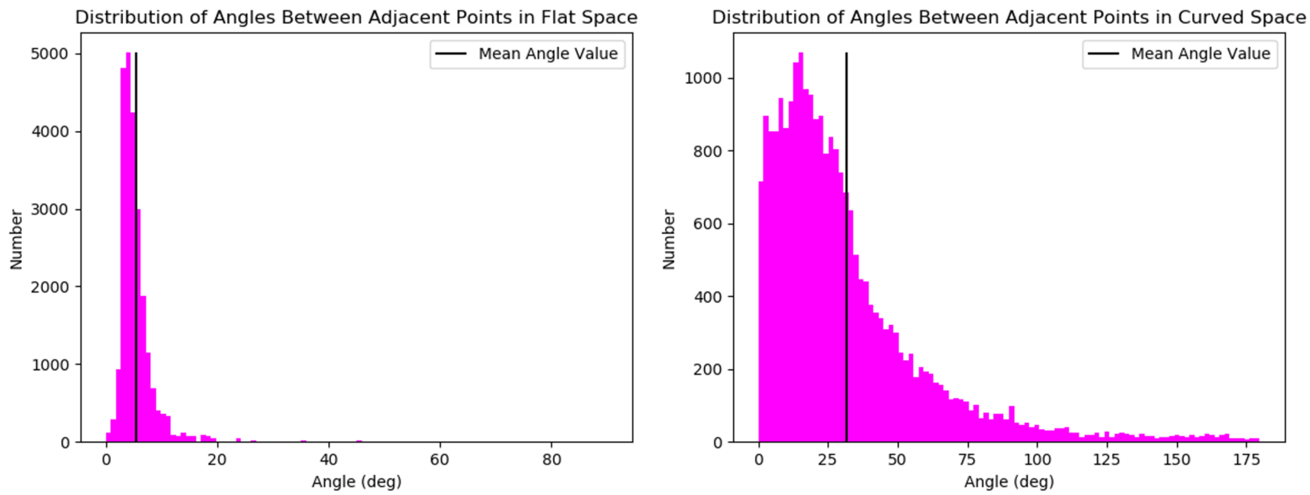

Figure 5. How the distribution of angles as well as the average angle value is affected by the spatial curvature. It is important to note that the exact results depend on the curvature functions used.

Figure 5 shows how the same curvature that was applied to produce Figure 4 affects the observed angles between previously adjacent pairs. In flat space, the angles between pairs is a very narrow distribution. This is expected due to the uniform nature of the points. When curvature is applied, the distribution loses the initial uniformity and the average angle between previously adjacent points will change. Generally this will be observed in a spreading of the distribution and an increase in the mean angle. For the test case here, the distribution spreads out and the mean angle increases.

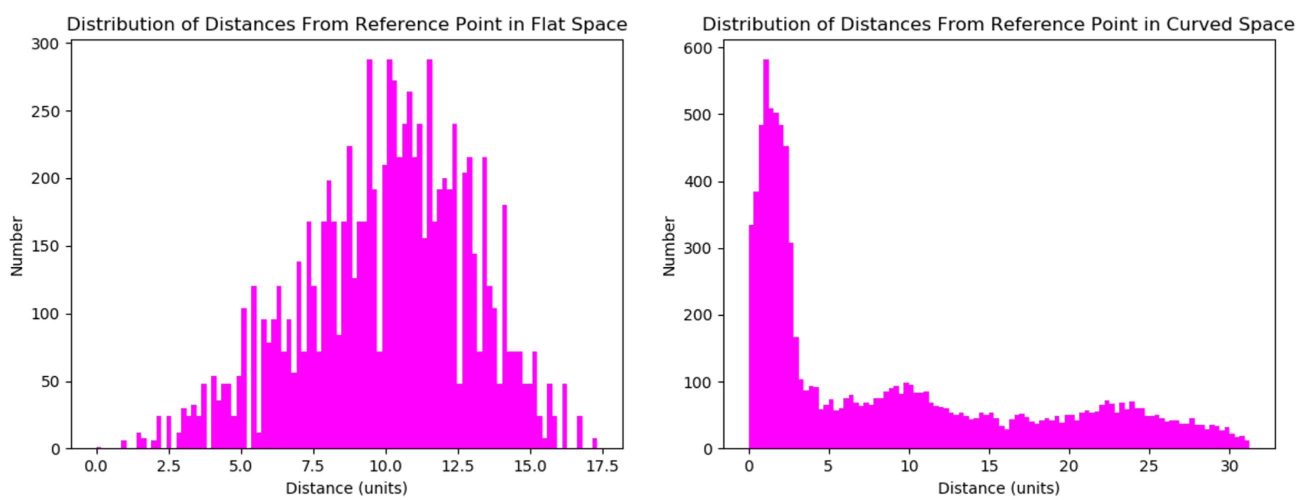

Figure 6. How the distribution of distances as well as the average distance value is affected by the spatial curvature. It is important to note that the exact results depend on the curvature functions used.

The last observable to examine is the radial distance to the points. The curvature parameter is also a function of time which is described by this distance. Figure 6 displays how curvature can effect the apparent distance between a point and the reference point. The distance in flat space is a near symmetrical distribution representative of the initial cubic uniformity. When curvature is applied, the distances to the points no longer appears as symmetrical as the initial uniformity is lost.

The program now successfully considers the observables I had set out in Section 2: the distance to an object and the angle formed between two objects through the reference point. Now I address the future fitting aspect of the program.

5. FUTURE IMPLEMENTATIONS

At it's current stage, CURVFAM does not yet posses a finalized fitting component. Future versions of CURVFAM will include a fitter that allows for the interpretation of real-world observational data. The fitter will utilize the same top-down method of describing curvature in order to determine which curvature parameter best fits imported observable data. The difference is that rather than applying curvature to a set of uniform points, the observational data will represent the data points in curved space. The fitter is then tasked with finding the best curvature functions to produce the observed results. An additional component that will be added to future versions is the inclusion of general relativity. When considering universal curvature, general relativity plays a significant part in the determination of universal parameters such as density parameters and values such as the Hubble constant. As such, the fitter will take relativity into account and return a curvature fit as well as possible universal parameter values.

With the large amounts of data available for use publicly (Guillochon et al. 2017) as well as the possibility of future supernovae surveys, the ability to process large amounts of data with speed and accuracy is important. CURVFAM already demonstrates the ability to handle thousands of data points in this current top-down state. However, with the addition of the fitting algorithm, a decrease in efficiency is to be expected. As such, new methods of modeling as well as new fitting algorithms will be continually tested and applied to CURVFAM.

In its current state, CURVFAM works as a client side code that must be directly modified to include the local data files. In future implementations we will apply a more user friendly front end GUI to help streamline data input and output. This will allow for the code to be accessed as either a package or program without the need to directly edit the source code. Stemming off from this, even further versions of CURVFAM can be modified to be presented in a web-based format. This will allow for the intense calculations to be run on more efficient servers and provide additional ease-of-use for the user.

6. CONCLUSION

In this paper I presented the current state of a continuing process towards unifying computational modeling and universal curvature fitting. CURVFAM is a code that allows for the determination of observables in a prescribed curved space. The implication of this is the ability to reverse this process and find a curvature that generates the observed curved space values. With the advent of a universe simulator designed with the sole purpose of determining curvature parameters, future curvature measurements with new and old data alike will be faster and less computationally intense.

In its current state, CURVFAM has the ability to display the effects of various curvatures on any chosen selection of data. This is useful in the testing of theories regarding the effects of curvature on systems. Later versions of CURVFAM will include a statistical fitting algorithm as well as a more user friendly GUI for streamlined data input and output. This will allow for the automated fitting of curvature parameters to large quantities of data. In the future, CURVFAM aims to generate a better model of the universe and increase our understanding of its topology both in space and in time.

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Software: Python

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